AGITATION OF A LIQUID IN A STRONG EXTERNAL ELECTRIC FIELD

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This article considers the condition for instability of the surface waves in a liquid system in a strong external electric field. As a result of the viscosity, such surface waves lead to vigorous volume agitation, accelerating chemical and physicochemical processes occurring in the system.

Heterogeneous electric fields have recently come into use for vigorous agitation in industrial physicochemical or chemical processes. Varying heterogeneous electric fields make it possible to create wave-like motions at the surface of a liquid, which, through internal-friction forces, produce powerful turbulent flows within the liquid and thus promote good agitation. This permits intensification of many processes that require agitation, including extraction, emulsion breakdown, synthesis of certain important compounds (hydrazine, nylon, etc.), and solution of solids in liquids.

If one places a rack of electrodes connected to a quasivarying high voltage source at the surface of the liquid (homogeneous or multiphase, conductive or nonconductive) and another electrode, immersed in the liquid, is grounded (Fig. 1), wave-like motions develop at the liquid surface and, under certain conditions, become so intense that they lead to vigorous volume agitation as a result of viscosity. There is no theory of this process, although practical applications of such an agitation method have been discussed in a survey [1].

It is therefore of great interest to consider the conditions that must be observed to produce vigorous liquid agitation under the action of an external electric field and to evaluate the energy consumed. It will be shown below that vigorous agitation occurs if the distance between the electrodes and the rack is equal to, or shorter than, the optimum wavelength, since resonance occurs in this case and the amplitude of the superficial waves is small. These optimum conditions depend on the critical electric-field strength and the latter in turn depends on the properties of the liquid (density, surface tension, kinematic viscosity, and dielectric permeability).

We will determine the conditions under which vibratory motions of a liquid in a heterogeneous, quasivarying external electric field are unstable and agitation is very vigorous. The relative motion of the phases in a local-gradient electric field will not be considered, so that we will deal only with the motion of the liquid as a whole, i.e., will consider the liquid or liquid system to be "quasihomogeneous."

Liquid motion can develop when the molecules move in the electric field in the direction of the greatest field gradient. The initial perturbation develops at the instant when the external constant or quasivarying electric field is applied.

The equation of motion for a "quasihomogeneous" incompressible viscous liquid is:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\nabla)\mathbf{v} = -\frac{1}{\rho}\operatorname{grad} p + \nu\Delta\mathbf{v} - \mathbf{g}. \tag{1}$$

At small motion velocities, $\partial v/\partial t \gg (v\nabla)v$. Taking this into account, Eq. (1) for the horizontal component y and vertical component z can be written in the form

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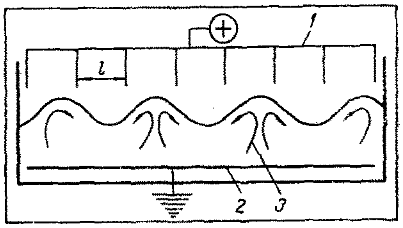

Fig. 1. Scheme of volume mixing of liquid: 1, 2) electrodes; 3) liquid; arrows show turbulent jets.

$$\frac{\partial v_y}{\partial t} = \nu\left(\frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2}\right) - \frac{1}{\rho}\frac{\partial p}{\partial y}, \tag{2}$$

$$\frac{\partial v_z}{\partial t} = \nu\left(\frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2}\right) - \frac{1}{\rho}\frac{\partial p}{\partial z} - g. \tag{3}$$

The discontinuity equation for an incompressible liquid is div $\mathbf{v} = 0$, or

$$\frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0. \tag{4}$$

We seek a solution to equation system (2)-(4) in the form of a fading plane wave (const exp kz + const exp nz)exp(− iωt + iky) (Re k, Re n > 0, with the z axis directed down into the liquid). The expression for the horizontal velocity component is

$$v_y = (\alpha \exp kz + \beta \exp nz) \exp(-i\omega t + iky). \tag{5}$$

Using Eq. (4), we write an expression for v_z:

$$v_z = -i\left(\alpha \exp kz + \frac{k}{n}\beta \exp nz\right) \exp(-i\omega t + iky). \tag{6}$$

Substituting Eq. (5) into equation of motion (2), we obtain

$$\left[i\omega\alpha e^{kz} + \nu\left(n^2 - k^2 - \frac{i\omega}{\nu}\right)\beta \exp nz\right] \exp(-i\omega t + iky) = \frac{1}{\rho}\frac{\partial p}{\partial y}. \tag{7}$$

If the relationship between n and k has the form

$$n = k\sqrt{1 - \frac{i\omega}{\nu k^2}}, \tag{8}$$

the preceeding equation can be rewritten in the form

$$i\omega\alpha \exp kz \exp(-i\omega t + iky) = \frac{1}{\rho}\frac{\partial p}{\partial y}. \tag{9}$$

Substituting Eq. (6) into Eq. (2) and taking into account Eq. (8), we find

$$\omega\alpha \exp kz \exp(-i\omega t + iky) - g = \frac{1}{\rho}\frac{\partial p}{\partial z}. \tag{10}$$

Joint solution of Eqs. (9) and (10) yields

$$\frac{p}{\rho} = -gz + \alpha\frac{\omega}{k}\exp kz \exp(-i\omega t + iky). \tag{11}$$

In order to determine the constants α and β, it is necessary to use boundary conditions: the normal and tangential pressures at the free boundary of the vibrating surface $\varkappa(y)$ equal 0.

An external quasivarying electric field exerts pressure on the surface of the vibrating liquid. The electric-field potential are

$$\varphi = \varphi_0 + \varphi_1 = -E_0 z + \text{const} \exp kz \exp(-i\omega t + iky)$$

over the vibrating surface ($z > 0$) and

$$\varphi' = \frac{1}{\varepsilon}(\varphi_0 + \varphi_1) = -\frac{E_0}{\varepsilon}z + \text{const} \exp(-kz) \exp(-i\omega t + iky)$$

beneath the vibrating surface ($z > 0$) and are sought in a form such that the perturbed term satisfies the Laplacian $\Delta\varphi_1 = 0$, $\Delta\varphi_1' = 0$ and disappears on moving away from the surface:

$$\varphi_1|_{z\to-\infty} \longrightarrow 0, \quad \varphi_1'|_{z\to+\infty} \longrightarrow 0.$$

Assuming that the displacement of the points on the vibrating surface is $\varkappa(y) = \text{const}\exp(-i\omega t + iky)$, we write

$$\begin{aligned} \varphi &= -E_0 z + E_0\varkappa(y)\exp(kz), \\ \varphi' &= -\frac{E_0}{\varepsilon}z - \frac{E_0}{\varepsilon}\varkappa(y)\exp(-kz). \end{aligned} \tag{12}$$

Using the boundary conditions for the electric-field potential (continuity of the normal component of the electric-field induction), the expression for the pressure has the form

$$\Delta p = \frac{1}{8\pi}\left[\left(\frac{\partial\varphi}{\partial n}\right)^2 - \left(\frac{\partial\varphi'}{\partial n}\right)^2\right]\Bigg|_{z=\varkappa} = \frac{E_0^2}{8\pi}\left(1 - \frac{1}{\varepsilon}\right)(1 - 2\varkappa k)$$

with an accuracy to terms of first order. We are interested only in the varying component of the electric-field pressure:

$$\Delta p' = -\frac{E_0^2}{4\pi}\varkappa k\left(1 - \frac{1}{\varepsilon}\right). \tag{13}$$

At the free-surface boundary $z = \varkappa(y)$ of the vibrating liquid, the normal pressure component is:

$$2\rho\nu\frac{\partial v_z}{\partial z} - \frac{\sigma}{R} + \Delta p' - p = 0, \tag{14}$$

and the tangential pressure component is

$$\rho\nu\left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right) = 0. \tag{15}$$

The radius of curvature of the surface is $R = \dfrac{\dfrac{\partial^2\varkappa}{\partial y^2}}{\left[1 + \left(\dfrac{\partial\varkappa}{\partial y}\right)^2\right]^{3/2}} \simeq \dfrac{\partial^2\varkappa}{\partial y^2}$. Taking into account Eqs. (5), (6), (11), and (13), Eqs. (14) and (15) yield

$$\left[g - \frac{E_0^2}{4\pi\rho}k\left(1 - \frac{1}{\varepsilon}\right)\right]\varkappa - \frac{\sigma}{\rho}\frac{\partial^2\varkappa}{\partial y^2} - \left[2ik\nu(\alpha\exp kz + \beta\exp nz) + \frac{\omega}{k}\alpha\exp kz\right]\exp(-i\omega t + iky) = 0, \tag{16}$$

$$2nk\alpha\exp kz = (k^2 + n^2)\beta\exp nz. \tag{17}$$

In first approximation, we can assume

$$\frac{\partial\varkappa}{\partial t} \simeq v_z. \tag{18}$$

Differentiating Eq. (16) with respect to time and taking into account Eqs. (6) and (18), we find

$$\left[g - \frac{E_0^2}{4\pi\rho}k\left(1 - \frac{1}{\varepsilon}\right) + \frac{\sigma k^2}{\rho} - 2i\nu k\omega - \frac{\omega^2}{k}\right]\alpha\exp kz + \left[\frac{k}{n}\left(g - \frac{E_0^2 k}{4\pi\rho}\left(1 - \frac{1}{\varepsilon}\right) + \frac{\sigma k^2}{\rho}\right) - 2i\nu k\omega\right]\beta\exp nz = 0. \tag{19}$$

On the basis of the compatibility condition for the solutions to Eqs. (17) and (19), taking into account Eq. (7), we write the dispersion equation for determination of the natural vibration frequency for the liquid:

$$\frac{\left[g - \dfrac{E_0^2}{4\pi\rho}k\left(1 - \dfrac{1}{\varepsilon}\right) + \dfrac{\sigma k^2}{\rho}\right]}{\nu^2 k^3} + \left(2 - \frac{i\omega}{\nu k^2}\right)^2 - 4\sqrt{1 - \frac{i\omega}{\nu k^2}} = 0. \tag{20}$$

At $\operatorname{Im}\omega > 0$, the system vibrations are unstable, because of the unlimited increase in vibration amplitude with time.

Let us consider the behavior of the system at low viscosities, where $|L_0| \gg \nu^2 k^3$. In this case, we find from Eq. (20) that

$$\omega = \pm\sqrt{L_0 k - 4\nu^2 k^4} - 2i\nu k^2, \tag{21}$$

where

$$L_0 = g - \frac{E_0^2}{4\pi\rho} k \left(1 - \frac{1}{\varepsilon}\right) + \frac{\sigma k^2}{\rho}. \tag{22}$$

At low electric-field strengths ($L_0 > 0$) the surface vibrates at a frequency

$$\omega = \pm \left(\sqrt{L_0 k} - \frac{2\nu^2 k^4}{\sqrt{L_0 k}}\right) - 2i\nu k^2.$$

At higher electric-field strengths ($L_0 < 0$),

$$E_0^2 > \frac{\varepsilon}{\varepsilon - 1}\left(4\pi\sigma k + \frac{4\pi g\rho}{k}\right) \tag{23}$$

motion is unstable, since the vibration amplitude increases with time by the rule

$$\exp\left[\operatorname{Re}(-i\omega t)\right] = \exp\left[\left(\sqrt{L_0 k} + \frac{2\nu^2 k^4}{\sqrt{L_0 k}}\right) t\right].$$

For large viscosities, where $|L_0| \ll \nu^2 k^3$,

$$\omega = \pm i \sqrt{\frac{4}{9}\nu^2 k^4 - \frac{2}{3} L_0 k} - \frac{2}{3} i\nu k^2.$$

At small electric-field strengths ($L_0 > 0$) the vibrations have an aperiodic charactor, with the frequencies

$$\omega_1 = -\frac{4}{3} i\nu k^2, \quad \omega_2 = -\frac{L_0 i}{2\nu k};$$

we are interested in ω_2, which is the smallest quantity. At high electric-field strengths ($L_0 < 0$), the vibrations have an aperiodic character, with slowly increasing amplitude

$$\omega_1 = \frac{|L_0| i}{2\nu k}$$

and slowly decreasing amplitude

$$\omega_2 = -\frac{4}{3}\nu k^2 i.$$

For large viscosities, attenuation at the frequency ω_2 is dominant since $|\omega_1/\omega_2| \ll 1$.

Let us analyze the behavior of the system at $|L_0| \sim \nu^2 k^3$. After transformation, Eq. (20) yields

$$z^4 + 2Az^2 - 4z + A^2 + 4 = 0, \tag{24}$$

where

$$\frac{L_0}{\nu^2 k^3} = A,$$

$$z = 2 - \frac{i\omega}{\nu k^2} \quad \text{or} \quad \omega = i\nu k^2 (z - 2). \tag{25}$$

It can be seen from Eq. (25) that the vibrations are unstable at $\operatorname{Re} z > 2$, because of the unlimited increase in amplitude with time. At $\operatorname{Re} z_{\mathrm{cr}} = 2$ (corresponding to A_{cr}), the vibrations are nonfading. In order to determine A_{cr}, we must solve Eq. (24). Using the Euler method [2], solution of Eq. (24) reduces to solution to two second-order equations and one cubic resolvent equation,

$$y^3 + 4Ay^2 - 16y - 16 = 0. \tag{26}$$

The roots of Eq. (24) are

$$\begin{aligned} z_{1,2} &= \frac{1}{2}\left[\sqrt{y_0} \pm \left(\sqrt{y_1} + \sqrt{y_2}\right)\right], \\ z_{3,4} &= \frac{1}{2}\left[-\sqrt{y_0} \pm \left(\sqrt{y_1} - \sqrt{y_2}\right)\right], \end{aligned} \tag{27}$$

where y_0, y_1, and y_2 are the roots of Eq. (26).

TABLE 1. Critical Parameters for Different Substances

Substance	ε	cm^2/sec	erg/cm^2	g/cm^3	kV/cm	kV/cm	λ_0,cm	λ_ν, cm
Benzene	2,28	0,00735	30	0,88	26,5	26,5	1,16	1,16
Glycerol	43	6,8	63,4	1,26	25,6	42,5	5,1	5,25
Water	80	0,01	72	1,0	24,8	24,8	1,7	1,7
Ethanol	33	0,015	24,1	0,79	17,6	17,6	1,1	1,1
Toluol	2,39	0,0067	24,8	0,87	24,0	24,0	1,14	1,14
Mercury	—	0,0012	472	13,55	75,0	75,0	1,18	1,18

Using the Cardano method to solve Eq. (26) for different numerical values of A, we find that $\mathrm{Re}\, z_{\mathrm{cr}} = 2$ at $A_{\mathrm{cr}} = -5.75$. Taking this into account, we find from Eq. (22) that the system vibrations are unstable at

$$E_0^2 > \frac{\varepsilon}{\varepsilon - 1}\left(23\pi\rho\nu^2 k^2 + \frac{4\pi\rho g}{k} + 4\pi\sigma k\right). \tag{28}$$

It is of interest to determine the least (critical) external electric-field strength at which the superficial vibrations become unstable. From the minimum condition for Eq. (28), we find

$$k^3 + \frac{\sigma}{11.5\rho\nu^2}\left[k^2 - \frac{\rho g}{\sigma}\right] = 0. \tag{29}$$

A solution to cubic Equation (29) can be found by the Cardano method. Since analytic solutions found by the Cardano method are cumbersome, we will first find the minimum (critical) wave number k_0 at a small viscosity ($\nu \sim 0$). In this case, Eq. (29) yields

$$k_0 = \sqrt{\frac{g\rho}{\sigma}}.$$

The critical electric-field strength, taking into account the foregoing, is given by the relationship

$$E_0^2 > \frac{\varepsilon}{\varepsilon - 1} 8\pi \sqrt{\rho g \sigma}. \tag{30}$$

As an example of interest for practical application, we will consider a system with the parameters $\varepsilon \sim 3$, $\rho \sim 1$ g/cm^3, and $\sigma \sim 16$ erg/cm^2. Using the above relationship, we find that the system is unstable at $E_0 > 20$ kV/cm, $k_0 \simeq 7.8$ cm^{-1}, and $\lambda_0 \simeq 0.8$ cm. The presence of viscosity obviously leads to an increase in the critical field strength E_0. At small viscosities, Eq. (29) can be used to find the approximate critical wave number. In this case, Eq. (29), taking into account those quantities of first order of smallness, yields

$$k_\nu = \left(\sqrt{\frac{g\rho}{\sigma}} + 5.75\frac{g\rho^2\nu^2}{\sigma^2}\right). \tag{31}$$

The critical electric field strength, taking into account Eq. (31), is given by the relationship

$$E_\nu > \frac{\varepsilon}{\varepsilon - 1}\left(8\pi\sqrt{\rho g \sigma} + 23\pi\frac{\rho^2\nu^2 g}{\sigma}\right). \tag{32}$$

A system with the parameters $\varepsilon \sim 3$, $\rho \sim 1$ g/cm^3, $\sigma \sim 16$ erg/cm^3, and $\nu \sim 3.3$ stk is unstable when $E_\nu > 22$ kV/cm.

A similar method was used to calculate the critical electric-field strength for different substances. The calculation results are given in Table 1.

The advantages of this method over mechanical agitation include more uniform agitation over the mixture volume, since there are no stagnant zones, and lower energy consumption, since the electric-field energy is directly converted to the mechanical energy of agitation.

In conclusion, it should be noted that our solution to the Navier–Stokes equation for surface waves pertains to a purely external flow regime and the sufficient conditions for flow turbulization in the liquid volume naturally cannot follow directly from this solution. However, it has been shown experimentally [1] that turbulent agitation of a liquid occurs in a strong external electric field; taking this into account, the instability conditions for surface waves can be used for analytic derivation of the necessary conditions for surface waves, with the aid of viscosity forces, to produce turbulent agitation within a liquid.

NOTATION

v	is the velocity;
ρ	is the density;
ν	is the kinematic viscosity;
g	is the gravitational acceleration;
p	is the hydrostatic pressure;
α, β	are the constants;
E_0	is the strength of external quasi-variable uniform electric field;
ε	is the dielectric permeability;
σ	is the surface tension coefficient for liquid;
R	is the radius of curvature of liquid surface.

LITERATURE CITED

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